Accurate Synthesis of Inline Prototype Filters Using Cascaded Triplet and Quadruplet Sections

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***Abstract*—A general method for designing low-pass prototype networks with cascade inline topology is presented. The generalized Chebyshev characteristic is imposed, allowing an equiripple response in the passband and transmission zeros arbitrarily displaced in the complex plane (symmetry with respect to the imaginary axis is, however, required). A computationally efficient method is employed for evaluating reflection and transmission polynomials, given the transmission zeros; then, the synthesis of a starting coupling matrix is performed using one of the methods available in the literature. The desired inline topology is then assembled by cascading elementary building blocks (triplets and/or quadruplets), following specific rules which assure the feasibility of the structure; finally, the synthesis is performed by applying a multiple similarity transform to the starting coupling matrix (the set of rotation angles is determined by means of an efficient and fast optimization procedure). Examples are reported which validate the proposed synthesis procedure.**

***Index Terms*—Circuit synthesis methods, coupling matrix, microwave filters.**

I. INTRODUCTION

SEVERAL works have recently addressed the synthesis of inline prototypes obtained by cascading triplet and/or quadruplet sections [1]–[5]. The various solutions proposed suffer, however, from little flexibility and/or accuracy: in fact, either they are applicable only to a few specific topologies, or the synthesis is performed through optimization of the frequency response, whose convergence is not *a priori* guaranteed.

In this paper, a novel procedure is presented for computing the coupling matrix of an arbitrary inline low-pass prototype, exhibiting a generalized Chebyshev response; the procedure starts with the synthesis either of the canonical folded prototype [6] or of a generic coupling matrix obtained with orthonormalization techniques [7], [8]. The coupling matrix of the inline prototype is then determined using multiple similarity transforms, whose unknown parameters (rotation angles) are computed through optimization. Let us observe that, in the proposed method, the optimization does not affect the frequency response of the synthesized network; in fact, it is known that starting from a generic admissible coupling matrix and assuming an inline prototype topology physically compatible with the imposed response, it is, in general, possible to find a sequence of similarity transforms determining the coupling matrix of the inline prototype [10]. Independently of how this sequence and the corresponding rotation angles are determined, the response of the transformed network is exactly the same as the original one.

The feasibility of various inline topologies obtained by cascading triplet and quadruplet sections is also discussed in the paper; in particular, specific rules are recalled, which must be followed for assembling a physically realizable prototype.

Finally, examples are presented, which illustrate and validate the synthesis procedure developed in the paper.

II. COUPLING MATRIX FOR THE GENERIC PROTOTYPE

Let us assume that a low-pass prototype exhibits the generalized Chebyshev response [8], [9] given by

$$A(\Omega^2) = \frac{1}{|S_{21}(j\Omega)|^2} = 1 + \varepsilon^2 C_N^2(\Omega) \tag{1}$$

where ε depends on the required passband ripple and C_N is the generalized Chebyshev function

$$\cos\left[(N-N_z)\cos^{-1}(\Omega) + \sum_{k}^{1,\,Nz}\left|\mathrm{Re}\left\{\cos^{-1}\left(\frac{1-\Omega\cdot\Omega_{z,\,k}}{\Omega-\Omega_{z,\,k}}\right)\right\}\right|\right], \qquad |\Omega| \le 1 \tag{2}$$

$$C_N(\Omega) = \cosh\left[(N-N_z)\cosh^{-1}(\Omega) + \sum_{k}^{1,\,Nz}\left|\mathrm{Re}\left\{\cosh^{-1}\left(\frac{1-\Omega\cdot\Omega_{z,\,k}}{\Omega-\Omega_{z,\,k}}\right)\right\}\right|\right], \quad |\Omega| > 1$$

where N is the order of the filter (number of poles), and N_z is the number of transmission zeros, whose values are $j\Omega_{z,k}$. Given the transmission zeros $\Omega_{z,k}$ and ε, various methods exist for evaluating the characteristic polynomials $E(s)$, $F(s)$, $P(s)$, defining the scattering parameters of the prototype [8], [9]

$$S_{21} = \frac{P(s)}{E(s)} \quad S_{11} = \frac{F(s)}{E(s)}. \tag{3}$$

Once the scattering parameters of the prototype are expressed as a ratio of polynomials, the synthesis of a generic prototype network can be faced. Direct methods are, however, known only for a limited number of topologies; one of these is the canonical folded prototype, whose general scheme is illustrated in Fig. 1.

Note that, even if a generalized Chebyshev response would allow a maximum number of zeros equal to $N-1$, for this

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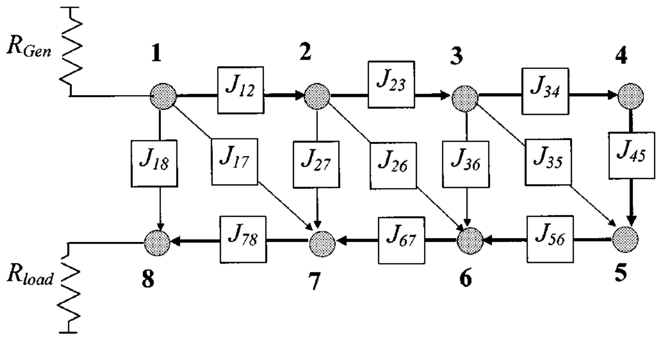

Fig. 1. Canonical cross-coupled prototype network. Each node represents a unit capacitance in parallel with a frequency invariant susceptance b_i; J_{ij} are the admittance inverters parameters between resonators i and j; $R_{\rm gen}$ and $R_{\rm load}$ are generator and load resistances.

prototype network it must be $N_z < N - 2$ because the cross couplings do not include generator and/or load.

The coupling matrix $\boldsymbol{J}$ for the canonical prototype is defined as

$$\mathbf{J} = \begin{matrix} b_1 & J_{1,2} & \cdot & \cdot & J_{1N} \\ J_{2,1} & b_2 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ J_{N,1} & \cdot & \cdot & \cdot & b_N \end{matrix} \tag{4}$$

where b_i are frequency-invariant susceptances in parallel to the unit capacitances at each node. Note that the elements J_{ij} referring to nodes i and j not connected through an admittance inverter are zeros.

Another method for obtaining a generic prototype is that originally proposed by Atia *et al.* [1] and recently revised by Cameron [8]. In this case, the coupling matrix (4) is numerically determined with an orthonormalization procedure that gives, in general, all the elements J_{ij} different from zero (this means that all the possible couplings between the nodes are present). This prototype is obviously not useful from a practical point of view, but it is a suitable starting point for further topologic transformations.

III. Evaluation of the Coupling Matrix for the Inline Prototype

Once an initial coupling matrix $\mathbf{J}$ has been synthesized using one of the above-mentioned methods, similarity transforms (also called rotations) can be applied; in this way, other equivalent coupling matrices can be derived, each presenting the same electrical response as the original coupling matrix, but with a different topology. This latter can be controlled by requiring that some elements of the transformed matrix vanish (note that the symmetry of $\mathbf{J}$ must be preserved). Some methods may be found in the literature which describe how to find the sequence of rotations (and the corresponding angles) required for obtaining a few specific topologies [8], [10]; however, a general procedure for an arbitrary topology it is not yet available. In this paper, a novel numerical method, based on multiple rotations, is described, which allows obtaining arbitrary topologies (once their feasibility has been assessed).

It is known [7] that a similarity transform of the matrix $\mathbf{J}$ of order N is defined by $\mathbf{R}_{ij}(\vartheta)\cdot\mathbf{J}\cdot\mathbf{R}_{ij}^t(\vartheta)$, where $\mathbf{R}_{ij}(\vartheta)$ is the rotation matrix of order N, pivot (i, j), and angle ϑ, defined as follows:

$$\begin{aligned} R_{ij}(i,\,i) &= R_{ij}(j,\,j) = \cos(\vartheta), \\ R_{ij}(i,\,j) &= -R_{ij}(j,\,i) = \sin(\vartheta) \\ R_{ij}(k,\,k)|_{k\neq i,\,j} &= 1, \qquad (i<j)\neq 1,\,N \\ R_{ij}(k,\,i)|_{k\neq i,\,j} &= 0, \qquad R_{ij}(j,\,k)|_{k\neq i,\,j} = 0. \end{aligned} \tag{5}$$

The conservation of the transfer function is also true for subsequent applications of the above transformation; therefore, we assume that the transformed matrix $\mathbf{J}'$ can be expressed as

$$\begin{aligned} \mathbf{J}' &= (\mathbf{R}_{23}\cdot\mathbf{R}_{24}\cdots\mathbf{R}_{N-2,\,N-1}) \\ &\quad \cdot\mathbf{J}\cdot\left(\mathbf{R}_{N-2,\,N-1}^t\cdot\mathbf{R}_{N-3,\,N-1}^t\cdots\mathbf{R}_{23}^t\right) \\ &= \mathbf{S}(\vartheta_1,\,\vartheta_2,\,\ldots\vartheta_M)\cdot\mathbf{J}\cdot\mathbf{S}^t(\vartheta_1,\,\vartheta_2,\,\ldots\vartheta_M) \end{aligned} \tag{6}$$

where M is the overall number of distinct pivots (i, j), for a given order N; M may be considered as the maximum number of independent rotations, and it is given by $M = (N^2 - 5N + 6)/2$. Let us now assume that a realizable topology has been specified for the transformed prototype; then, a set of rotation angles $(\vartheta_1,\,\vartheta_2,\,\ldots\vartheta_M)$ can be searched numerically for which the elements of $\mathbf{J}'$ corresponding to the cross couplings not included in the new topology are zeros. In fact, these angles must satisfy the following system of nonlinear equations:

$$J'_{k,\,l}(\vartheta_1,\,\vartheta_2,\ldots\vartheta_M) = 0 \tag{7}$$

where k and l span over all the elements of $\mathbf{J}'$ which must vanish.

The numerical solution of the above system may be performed through the minimization of a nonlinear cost function U defined as

$$U = \sum_{k.l} \left|J'_{kl}(\vartheta_1,\,\vartheta_2,\ldots\vartheta_M)\right|^2. \tag{8}$$

In the practical implementation of the minimization procedure, the Gauss–Newton method has proved to be fast, accurate, and little sensitive to the starting point. Note that, although it cannot be demonstrated theoretically that M independent rotations are sufficient to find an arbitrary (realizable) topology, for the several inline topologies that have been tested in this work, the developed procedure has always found the required transformation.

It should also be noted that the optimization procedure does not affect the accuracy of the inline prototype response, provided the final value of the residue [U in (8)] is sufficiently small (typically, $<\approx 10^{-10}$). A larger residue means that either the imposed topology is incompatible with the frequency response of the original synthesized coupling matrix, or the optimization algorithm is unable to find the optimum angles; in both cases, the response of the network obtained is, in general, unacceptable. Some difficulty in fulfilling the above target may arise when the order N of the filter become very large. Note, however, that practical applications of the considered prototypes rarely

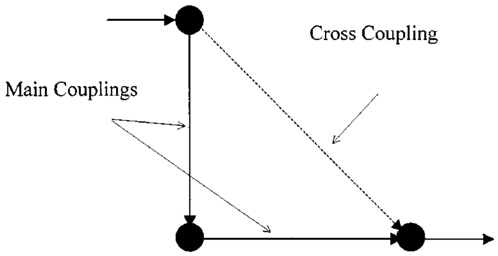

Fig. 2. Triplet structure.

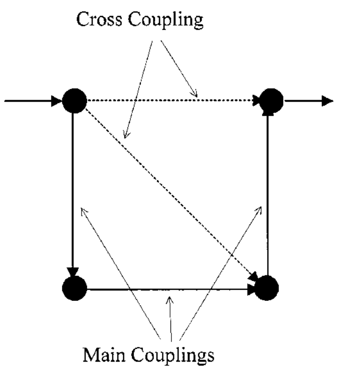

Fig. 3. Quadruplet structure. The oblique cross coupling could be displaced also along the other diagonal and it is not required in the case of symmetric imaginary zeros.

require an order larger than 12; in this case, the method here proposed works very well, as will be shown in Section V.

IV. Construction of the Inline Topology

Let assume now that the inline prototype to be synthesized is assembled as a cascade of triplet or quadruplet sections. Some rules must be then observed when performing this operation, in order to obtain a realizable network (i.e., compatible with the transmission characteristic imposed). These rules can be described as follows.

- Triplets (Fig. 2) realize imaginary transmission zeros (attenuation poles) asymmetrically displaced along the $j\Omega$ axis. For positive zeros, the cross coupling must be positive, while negative zeros require a negative cross coupling.
- Quadruplets (Fig. 3) realize pairs of transmission zeros, either imaginary or complex (with opposite real part); when the imaginary zeros are symmetric about the real axis, the oblique cross coupling is not necessary. Note that each transmission zero is influenced by both cross couplings.
- Triplets and/or quadruplets can be directly cascaded but can share only one node.

Note that only the location of the cross couplings must be given while their signs (positive or negative) result from the transformation procedure.

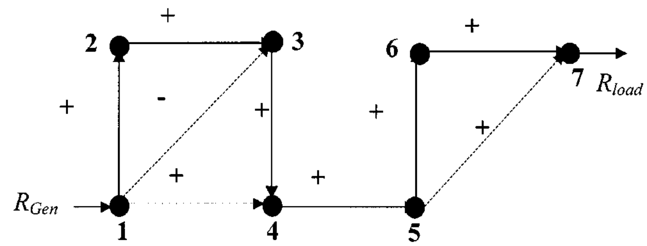

Fig. 4. Topology of the inline prototype to be synthesized.

V. Example

As an example of application of the inline prototype synthesis, let us consider the following electrical specifications:

- order N : 7;
- return loss: 23 dB;
- transmission zeros at $s_{1,2} = \pm 0.9218 - j0.1546$, $s_3 = j1.2576$.

Using the procedure developed in [9], the polynomials $N(s)$, $M(s)$, $P(s)$ are computed and, from these, the generic coupling matrix **J** is derived using the orthonormalization technique described in [8, Appendix I], as shown in the matrix at the bottom of this page.

Note that this matrix is not unique, and all its elements are, in general, different from zero. The generator and load resistances are, respectively, $R_{\text{gen}} = 0.8947$ and $R_{\text{load}} = 0.8945$.

Now, applying the novel procedure, the inline prototype illustrated in Fig. 4 is synthesized. The pivots defining all the independent matrix rotations are the following: {[2 3], [2 4], [2 5], [2 6], [3 4], [3 5], [3 6], [4 5], [4 6], [5 6]}; the optimization procedure previously described gives the following rotation angles (410 iterations are required with a final value for the residue $U = 8.2 \cdot 10^{-11}$):

$$\Theta' = [0.0999\ 2.0883\ -0.6833\ 0.3906\ 0.1530\ 2.2765\ -0.1191\ 0.5677\ -0.2930\ -2.8821].$$

0.0211	−0.7062	−0.1491	0.1985	0.2869	0.3827	−0.0000
−0.7062	0.2008	−0.1497	0.0957	0.4075	0.3446	−0.3835
−0.1491	−0.1497	0.2707	0.3027	0.2927	0.4556	0.1779
0.1985	0.0957	−0.3027	0.7223	0.2157	0.0269	0.3941
0.2869	−0.4075	0.2927	0.2157	0.0973	0.5604	−0.4204
0.3827	0.3446	0.4556	0.0269	0.5604	−0.0903	−0.5276
−0.0000	−0.3835	0.1779	0.3941	0.4204	−0.5276	0.0211

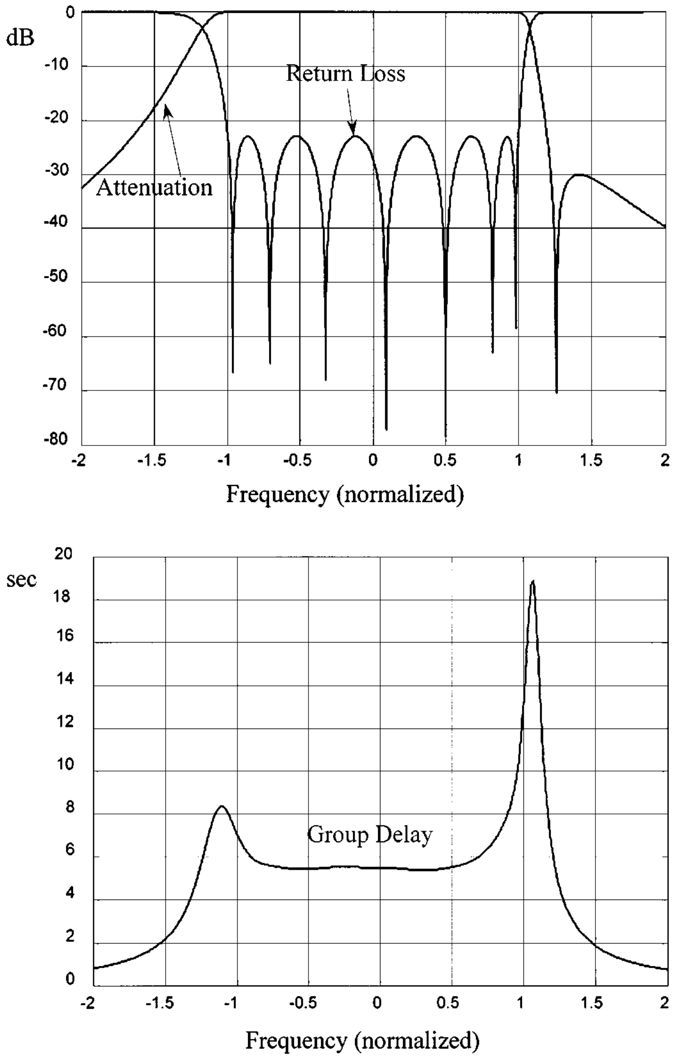

Fig. 5. Computed response of the synthesized inline prototype.

The new coupling matrix $\mathbf{J}'$ resulting after the matrix rotations is shown at the bottom of this page.

Fig. 5 shows return loss, attenuation, and group delay computed from the inline prototype; note that, while computing these responses, all the elements of $\mathbf{J}'$ defining couplings not present in the inline prototype have been set exactly to zero.

It has been observed in the previous section that the synthesis procedure here developed could fail in case the order N of the prototype becomes relatively large; moreover, the convergence may be also affected by the method employed for generating the initial coupling matrix $\mathbf{J}$.

TABLE I
CONVERGENCE OF THE SYNTHESIS PROCEDURE VARYING THE ORDER OF THE PROTOTYPE AND THE INITIAL COUPLING MATRIX

N	Canonical prototype		Ortho-norm. prototype	
	Iterations	$U \cdot 10^{13}$	Iterations	$U \cdot 10^{13}$
9	357	2.2	626	3.9
9	407	6.6	1279	0.44
9	522	0.1	1013	0.11
10	1026	0.24	1939	1
10	708	0.24	1800	0.97
10	1414	2.3	1413	5.3
11	886	0.44	1948	10
11	793	0.077	3617	190
11	510	160	2121	5
12	870	1.65	1899	0.14
12	871	0.02	4137	0.062
12	541	0.65	2935	12.3

In order to investigate these problems, several prototypes have been designed varying N (from 9 to 12), with a return loss of 25 dB and four transmission zeros randomly generated (they are obtained in the inline topology through two triplets and one asymmetric quadruplet); the initial matrix has been generated both through the canonical prototype synthesis and with the orthonormalization technique. Table I reports for each design performed the number of iterations and the value of the residue U at the end of the optimization (for both the initial coupling matrices).

The results in Table I show that the initial coupling matrix obtained from the canonical prototype allows a much faster convergence with respect to the other initial coupling matrix.

Note that all the prototypes synthesized produce a frequency response indistinguishable from the theoretical one [i.e., analytically evaluated from (1)]. Note also that the computer time required to perform 500 iterations on a Pentium III 600 MHz is about 3.2 s with $N = 12$.

VI. CONCLUSIONS

An original procedure for the synthesis of inline prototype filters based on multiple matrix rotations has been presented. The prototypes exhibit the generalized Chebyshev response, with arbitrarily placed transmission zeros, either purely imaginary or complex; the inline topology is obtained by cascading triplets and/or quadruplets sections, according to suitable rules presented in the paper. The accuracy of the proposed

$$\begin{array}{ccccccc}
0.0211 & 0.8583 & -0.0592 & 0.2211 & 0 & 0 & 0 \\
0.8583 & 0.1109 & 0.4734 & 0 & 0 & 0 & 0 \\
-0.0592 & 0.4734 & 0.0416 & 0.5855 & 0 & 0 & 0 \\
0.2211 & 0 & 0.5855 & 0.0417 & 0.5933 & 0 & 0 \\
0 & 0 & 0 & 0.5933 & 0.1057 & 0.4291 & 0.5659 \\
0 & 0 & 0 & 0 & 0.4291 & -0.7383 & 0.6848 \\
0 & 0 & 0 & 0 & 0.5659 & 0.6848 & 0.0211
\end{array}$$

synthesis method has been assessed through several designs using different values for the order of the prototypes and two different techniques for generating the starting coupling matrix.

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